(Pages : 4) **J – 1900**

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, March 2020

First Degree Programme under CBCSS

Mathematics

Core Course X — MM 1642 — LINEAR ALGEBRA

(2014 admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. **Each** carries **1** mark.

1. For which choice of x, will the following pair $\left(\begin{pmatrix} x \\ x^2 \end{pmatrix}, \begin{pmatrix} 1 \\ x \end{pmatrix}\right)$ be linearly dependent?

2. What is the equation of the line along the vector $\begin{pmatrix} -b \\ a \end{pmatrix}$?

3. Find the angle between $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

4. Find the projection of $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ to the line along $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

5. Find the distance from $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ to the line $y = 2x$.

6. Let P be the transformation which assigns to each vector X the projection of X on the line along the vector $u = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. If $X = \begin{pmatrix} x \\ y \end{pmatrix}$, find $P(X)$.

P.T.O.

7. Let $X = \begin{pmatrix} x \\ y \end{pmatrix}$, calculate $R_{\pi}(X)$.

8. Let P be the linear transformation of the plane, given by projection to the line along $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Write down the matrix of the linear transformation.

9. What are the eigen values of the identity transformation on the plane.

10. Let R_{θ} and R_{ϕ} be rotations by angles θ radians and ϕ radians respectively. Describe geometrically $R_{\theta} R_{\phi}$.

SECTION – II

Answer **any eight** questions from this section. **Each** question carries **2** marks.

11. Write an equation of the line through $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ parallel to the vector $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

12. Find all solutions to the system of equations

$$5x + y = 0$$
$$-10x - 2y = 0$$

13. Calculate $R_{\frac{\pi}{4}}(X)$ if $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

14. Check whether the transformation which send $\begin{pmatrix} x \\ y \end{pmatrix}$ into $\begin{pmatrix} x \\ x^2 + y \end{pmatrix}$ is a linear transformation.

15. Find a linear transformation which is equal to $R_{\frac{3\pi}{2}}$.

16. Let A be an arbitrary linear transformation. Prove that A sends the origin into the origin.

17. Let A the transformation with matrix $\begin{pmatrix} 2 & 3 \\ -1 & 0 \end{pmatrix}$. Find the image of the line along $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ under A.

18. Let B the reflection in the x-axis and A is the reflection in the y-axis. Find BA.

19. Let A be a linear transformation and let I denote the identity transformation. Prove that $AI = IA = A$.

20. Let H_k be the transformation with matrix $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$ show that $H_k H_m = H_{k+m}$.

21. Define elementary matrices with examples.

22. Construct a linear transformation with eigen values 3 and 4.

SECTION – III

Answer **any six** questions from this section. **Each** question carries **4** marks.

23. Let R_θ be the rotation transformation through angle θ radians counterclockwise. If $X - \begin{pmatrix} x \\ y \end{pmatrix}$ and $R_\theta(X) = \begin{pmatrix} x' \\ y' \end{pmatrix}$, prove that $\begin{aligned} x' &= (\cos\theta)x - (\sin\theta)y \\ y' &= (\sin\theta)x + (\cos\theta)y \end{aligned}$.

24. Prove that if A is a linear transformation in the plane, then $A(X + \overline{X}) = A(X) + A(\overline{X})$ for every pair of vectors $X, \overline{X}$.

25. Let P denote projection on the line along $u = \begin{pmatrix} u \\ v \end{pmatrix}$. Prove that $P(E_1) = \begin{bmatrix} \frac{u^2}{u^2+v^2} \\ \frac{uv}{u^2+v^2} \end{bmatrix}$

and $P(E_2) = \begin{bmatrix} \frac{uv}{u^2+v^2} \\ \frac{v^2}{u^2+v^2} \end{bmatrix}$ where $E_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $E_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. What is matrix of P?

26. Write a linear transformation in $\mathbb{R}^4$ and find the corresponding matrix.

27. Prove that the commutative law of multiplication does not hold for the product of linear transformations.

28. Let U be the linear transformation having matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

    (a) Interpret u geometrically

    (b) Show that $UU = I$.

29. Let L be a straight line through the origin and θ be the angle from the positive x-axis to L. Find the matrix of the transformation S which reflects each vector in L.

30. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ such that $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$. Prove that if π is any parallelogram with one vertex at 0, then area $(A(\pi)) = \left(\textit{absolute value of} \begin{vmatrix} a & b \\ c & d \end{vmatrix} \right)$ area (π).

31. Describe the curve $x^2 + 2xy + y^2 = 1$.

SECTION – IV

Answer **any two** questions from this section. **Each** question carries **15** marks.

32. Let A be a linear transformation and let L be a straight line. Prove that the image of L under A is either a straight line or a single point.

33. Prove that if A, B, C are three linear transformations, then $(AB)C = A(BC)$ and $(m(A)m(B)m(C) = m(A)(m(B)m(C)))$.

34. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Prove that if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$, then A preserves orientation and if $\begin{vmatrix} a & b \\ c & d \end{vmatrix} > 0$, then A does not preserve orientation.

35. Let A be a linear transformation with symmetric matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and let t_1, t_2 be distinct eigenvalues of A. Choose nonzero eigenvectors X_1, X_2 corresponding to t_1 and t_2 respectively. Prove that X_1 and X_2 are orthogonal.
